ETH zürich

D MAVT
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Technische Mechanik
151-0223-10

Vorlesung 05

Freiheitsgrad
Degree of freedom

Herbstsemester 2024

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Auf wie viele Weisen kann sich ein System bewegen?

Freiheitsgrad: *Die minimale Anzahl Koordinaten für die eindeutige Bestimmung der Lage eines bestimmten Systems*

$$f = n - b$$

f: Freiheitsgrad gebundenes Systems

f: degrees of freedom of constrained system

n: Freiheitsgrad ungebundenes Systems

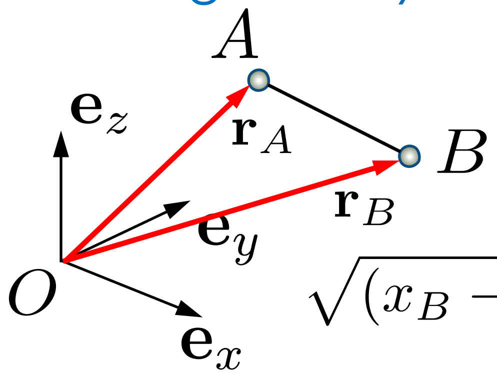
n: degrees of freedom of unconstrained system

b: Anzahl unabhängiger Bindungen

b: number of indipendent constraints

4.1 Starrkörper aus materiellen Punkten

4.1 Rigid body from material points



$$|\mathbf{r}_B - \mathbf{r}_A| = L$$

Konstanter Abstand zwischen zwei Punkten (Link): 1 skalare Gleichung (b=1)

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2} - L = 0$$

Freiheitsgrad ungebunden Punktes

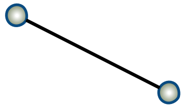
$$f = 2 \cdot 3 - 1 = 5$$

Punkte

Bindungen

Starrkörper in Ebene: f=3

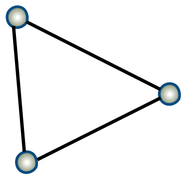
Ebene (2D)



$n = 2 \cdot 2 = 4$

$b = 1$

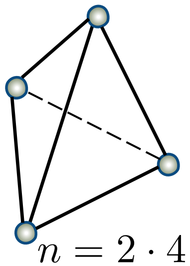
$f = n - b = 3$



$n = 2 \cdot 3 = 6$

$b = 3$

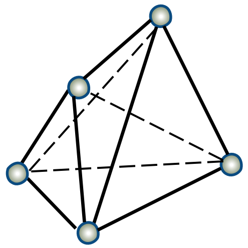
$f = n - b = 3$



$n = 2 \cdot 4 = 8$

$b = 5$

$f = n - b = 3$

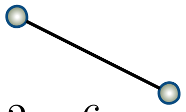


$n = 2 \cdot 5 = 10$

$b = 7$

$f = n - b = 3$

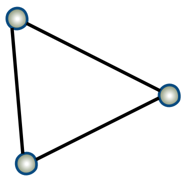
Raum (3D)



$n = 3 \cdot 2 = 6$

$b = 1$

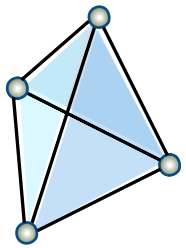
$f = n - b = 5$



$n = 3 \cdot 3 = 9$

$b = 3$

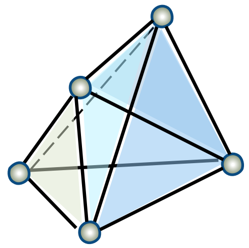
$f = n - b = 6$



$n = 3 \cdot 4 = 12$

$b = 6$

$f = n - b = 6$



$n = 3 \cdot 5 = 15$

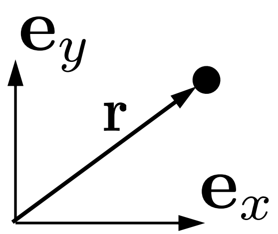
$b = 9$

$f = n - b = 6$

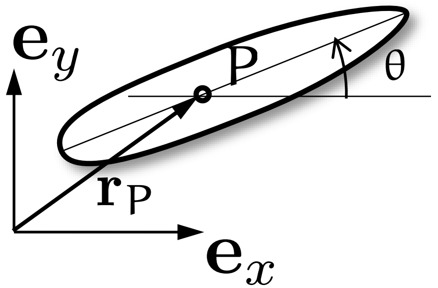
Starrkörper in Raum: f=6

4.2 Freiheitsgrad von freien Punkten und freien Starrkörpern

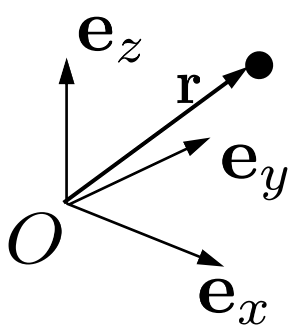
4.2 Degree of freedom of free points and rigid bodies



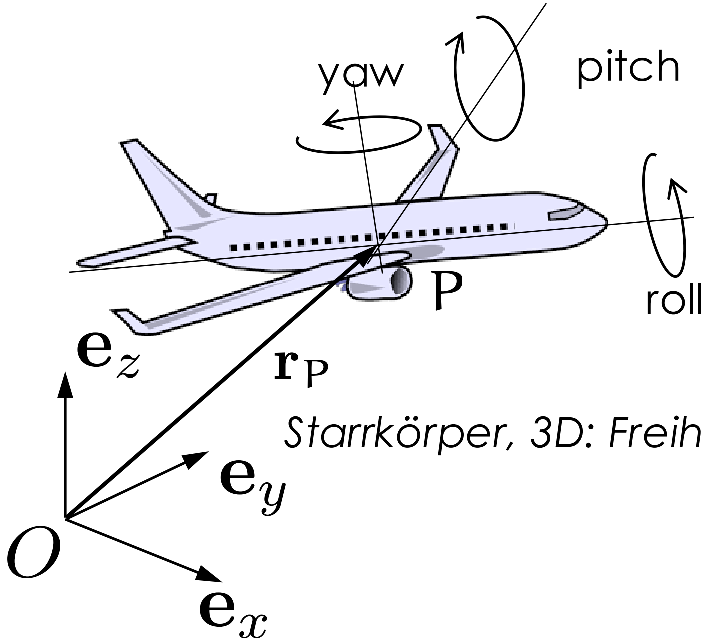
Materieller Punkt, 2D: Freiheitsgrad=2



Starrkörper, 2D: Freiheitsgrad=3



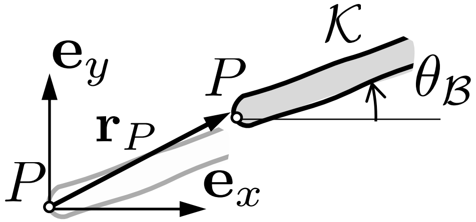
Materieller Punkt, 3D: Freiheitsgrad=3



Starrkörper, 3D: Freiheitsgrad=6

4.3 Bindungen in der Ebene

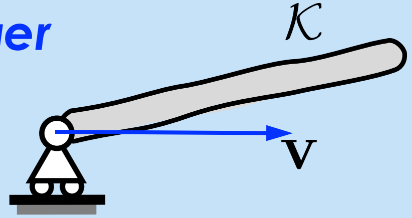
4.3 Planar constraints



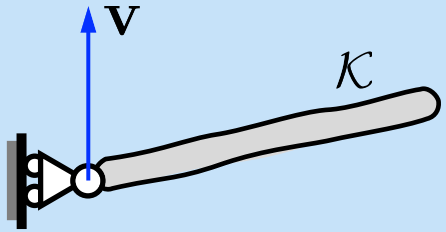
$$[\mathbf{r}_P] = [r_{Px}, r_{Py}]^T$$

Auflager
roller

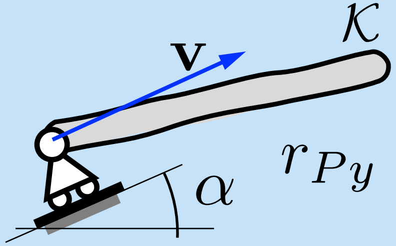
b=1



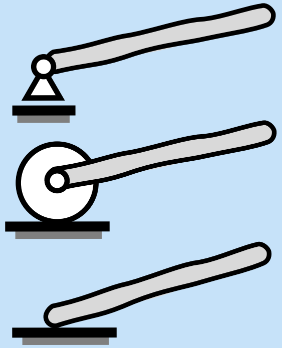
$$r_{Py} = 0$$



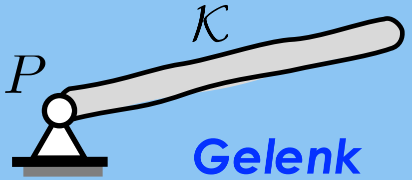
$$r_{Px} = 0$$



$$r_{Py} = r_{Px} \tan\alpha$$

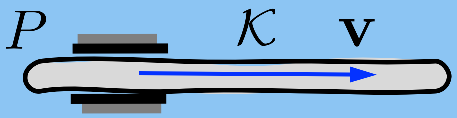


b=2



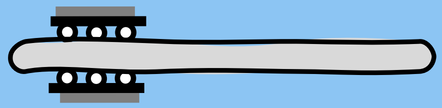
Gelenk
hinge

$$r_{Px} = 0$$
$$r_{Py} = 0$$

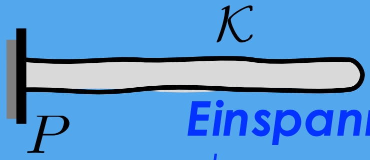


Slider

$$r_{Py} = 0$$
$$\theta_{\mathcal{K}} = 0$$



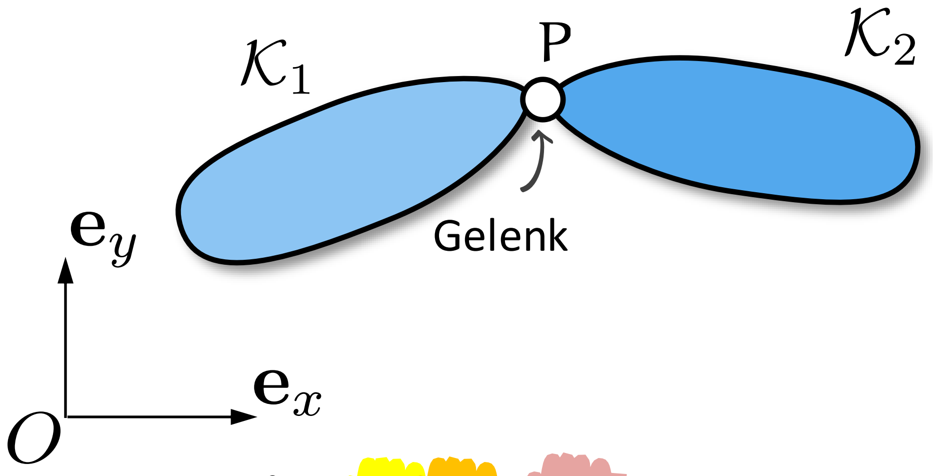
b=3



Einspannung
clamp

$$r_{Px} = 0$$
$$r_{Py} = 0$$
$$\theta_{\mathcal{K}} = 0$$

4.3 Bindungen in der Ebene

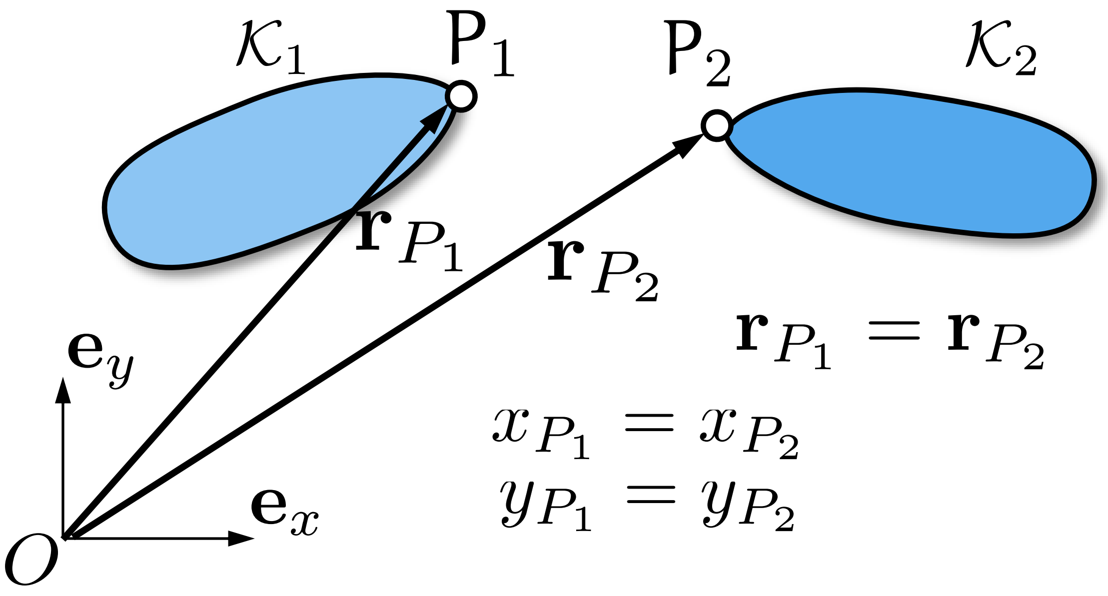


$$f = 2 \cdot 3 - 2 = 4$$

\# Starrkörper in 2D

\# Bindungen

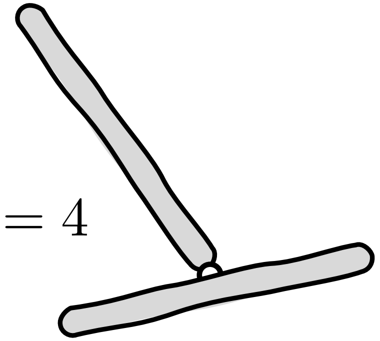
Freiheitsgrad ungebundenen SK in 2D

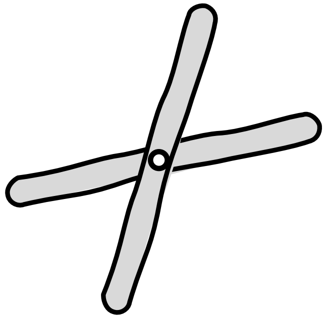


$$\mathbf{r}_{P_1} = \mathbf{r}_{P_2}$$

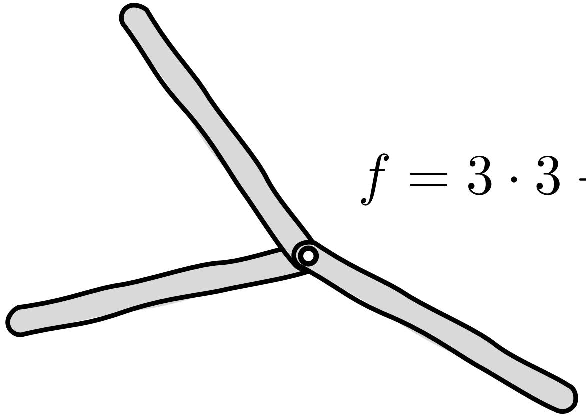
$$x_{P_1} = x_{P_2}$$
$$y_{P_1} = y_{P_2}$$

$$f = 2 \cdot 3 - 2 = 4$$





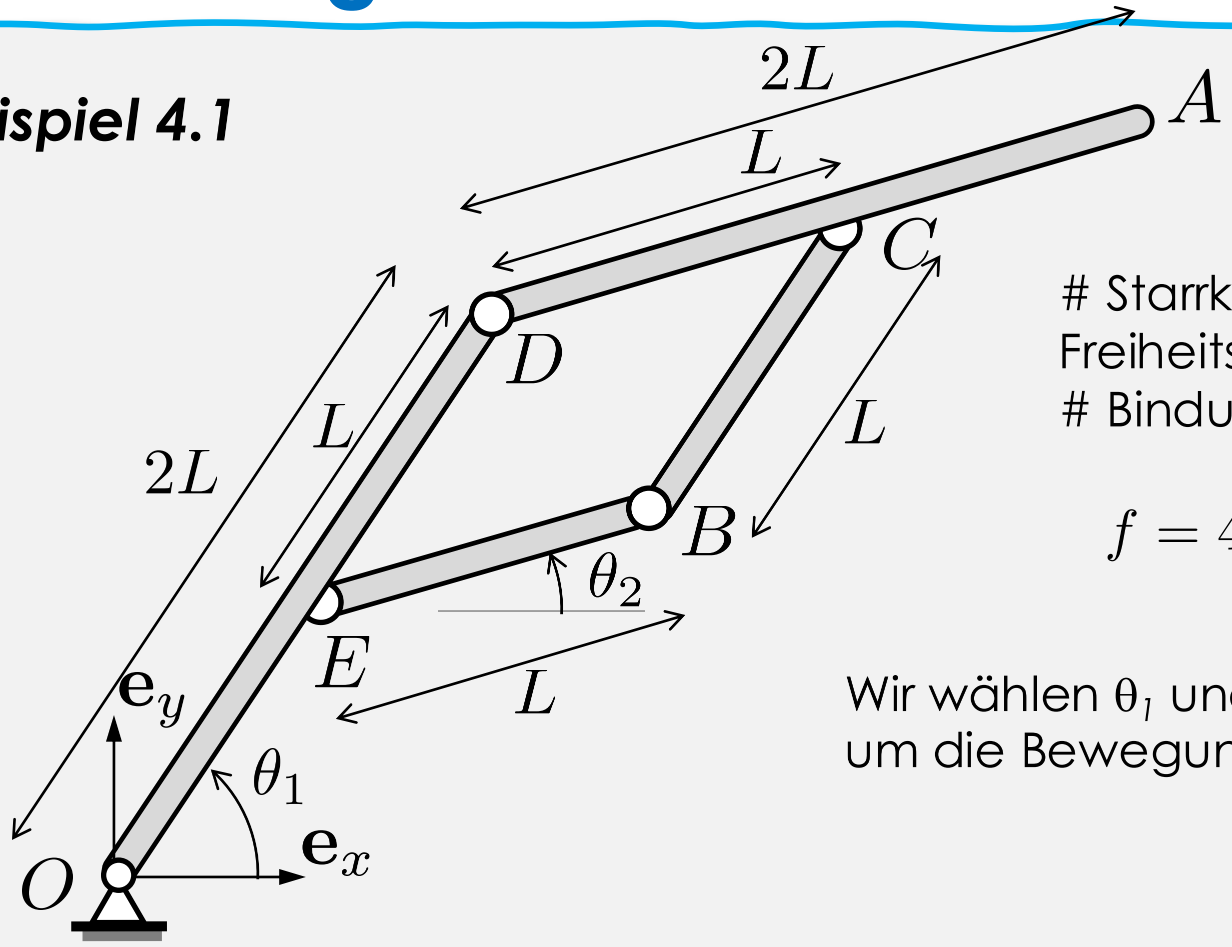
$$f = 2 \cdot 3 - 2 = 4$$



$$f = 3 \cdot 3 - 4 = 5$$

4.3 Bindungen in der Ebene

Beispiel 4.1



\# Starrkörper: 4
Freiheitsgrad ungebundenes Körper in 2D: 3
\# Bindungen: 10 (5 Gelenke)

$$f = 4 \cdot 3 - 10 = 2$$

Wir wählen θ_1 und θ_2 als allgemeine Koordinaten, um die Bewegung des Systems zu beschreiben.

$$\mathbf{r}_A = \mathbf{r}_D + \mathbf{r}_{DA} = (2L\cos\theta_1 + 2L\cos\theta_2)\mathbf{e}_x + (2L\sin\theta_1 + 2L\sin\theta_2)\mathbf{e}_y$$

$$\mathbf{r}_B = \mathbf{r}_B + \mathbf{r}_{EB} = (L\cos\theta_1 + L\cos\theta_2)\mathbf{e}_x + (L\sin\theta_1 + L\sin\theta_2)\mathbf{e}_y$$

$$\mathbf{r}_A = 2\mathbf{r}_B, \forall\theta_1, \theta_2$$